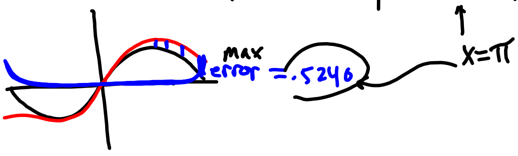
9.3 Taylor series with remainder

What is the fifth order Maclaurin series for f(x)=sin(x)? What is the maximum error when approximating sin(x) on $[-\pi,\pi]$

Solve graphically and numerically

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\text{error} = \left|\sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)\right|$$



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How many terms are needed in the Maclaurin series for sin(x) in order to approximate sin(x) within .0001 on the interval $[-\pi,\pi]$

max error

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}$$

too small

$x=\pi$

series

✓max error

✓interval

add one more term, worked

7 terms

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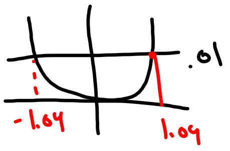
On what interval does the third order Maclaurin series approximate sin(x) within .01?

func

error

series

$$\text{error} = \left|\sin x - \left(x - \frac{x^3}{3!}\right)\right| \le .01$$



$[-1.04, 1.04]$

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Taylor's Remainder Estimation Theorem

$$\text{error} \le \left|\frac{M\,(x-c)^{n+1}}{(n+1)!}\right|$$

M is the max of $f^{(n+1)}(x)$ on the interval

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Ex 5 The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when x is small.

Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \le .01$ Support the answer graphically.

$-.01 \le x \le .01$

f'''

$-.01$ $.01$

$n=2$

$\text{error} \le \left| M \frac{x^3}{3!} \right|$

$\ln(1+x)$

$f' = \frac{1}{1+x}$

$M = \max \text{ of } |f^{(3)}(x)|$

$f'' = \frac{-1}{(1+x)^2}$

$M = \frac{2}{(1-.01)^3}$

$f''' = \frac{2}{(1+x)^3}$

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